

(12/18 Fri. 09 : 30 — 10 : 20)

The Mathematical Analysis and Data Forecast in COVID-19 Model with Applications

Jann-Long Chern

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Abstract: In this talk, we firstly propose two mathematical models including the diffusion effects, SQIARD and SIARD, to investigate the transmission of COVID -19 with quarantine, infected and asymptomatic infected, and discuss the relation between the respective basic reproduction number R_0 , $R_{\{Q\}}$ and the stability of the equilibrium points of model. Secondly, after training the related data parameters, in our numerical simulations, we respectively conduct the forecast of the data of US, South Korea, Brazil, India, Russia and Italy, and the effect of prediction of the epidemic situation in each country. Furthermore, we apply US data to compare SQIARD with SIARD , and display the effects of predictions.

(12/18 Fri. 10 : 20 — 10 : 50)

The sharp A_2 bound for Hilbert transform

Chih-Chieh Hung

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Abstract: In harmonic analysis, the sharp A_2 bound has been considered for a long time. This question was solved for Hilbert transform by S. Petermichl in 2007 using a quite hard technique, Bellman function method. Later, there are some new and simpler approaches to this question. In this talk, I will introduce a new approach given by E. Sawyer et al.

(12/18 Fri. 11 : 10 — 11 : 40)

Time-Frequency Analysis on Modulation Invariant Operators

Yung-Chang Hsu

National Taiwan University

Abstract: The theoretical application of Time-Frequency Analysis to operators in Harmonic Analysis dates back to 1973 when Charles Fefferman gave an alternate proof of the a.e. convergence of Fourier series of L^2 functions. This spurred the development of relevant ideas in the fields. For example, Christoph Thiele and Michael Lacey, inspired by Fefferman's idea, came up with their own approach and proved the boundedness of bilinear Hilbert transform in 1997.

As the latter technique developed, transformed, and prevailed, it had been applied to deal with problems revolving around linear modulation. However, it ran into a brick wall dealing with operators with higher-order modulation symmetry. One such example is the Polynomial Carleson Operator, which correlates to the understanding of some parabolic PDE and serves as a basic model for operators with implicit higher-order modulation symmetry. Stein conjectured the full range L^p bound of the operator, and Victor Lie proved exactly that in 2019 by rejuvenating Fefferman's idea and introducing a framework for analysis on polynomial modulation. Later on, Pavel Zorin-Kranich generalized the result to a higher dimensional setting through the same philosophy and simplified some of the constructions in Lie's arguments.

In this talk, aside from a brief history of the development, I will provide a heuristic explanation for Lie's and Zorin-Kranich's arguments, and, if time permits, we can discuss some interesting details.

(12/18 Fri. 11 : 40 — 12 : 10)

Keakeya conjecture and the projection theorem

Guo-Dong Hong

National Taiwan University

Abstract: The classical Keakeya problem was first posed by Soichi Keakeya in 1971, and it was disproved by Samoilovitch Besicovitch 11 years later; although this counter-example was similar to the solution of Riemann integration problem solved by Besicovitch himself in 1919. During the rapid development of measure theory, after the introduction of the notion about Hausdorff measure by Hausdorff in 1918, the modern Keakeya problem concerning the Hausdorff dimension was then asked.

In this talk, I will present two different approaches in fractal geometry and harmonic analysis respectively to the Keakeya problem.

In the first part, I will present two important projection theorems in fractal geometry originated and motivated again by Besicovitch to fully solve the Keakeya problem in $\mathbb{R}^2$.

On the contrary, the Keakeya problem in $\mathbb{R}^d$ when $d \geq 3$ is extremely difficult and has still remained as an open problem known as the Keakeya conjecture. So in the second part, I will present the use of maximal function in harmonic analysis in the work of Jean Bourgain in 1991, in which he integrated the idea before and reduced the Keakeya problem to the bound of the Keakeya maximal function introduced by Antonio Cordoba in 1977.

Finally, I will discuss some recent progress on the Keakeya problem; namely, the improved bound of the Keakeya maximal function.

(12/18 Fri. 13 : 10 — 14 : 00)

Introduction to Flag Hardy spaces

Ming-Yi Lee

National Central University

Abstract: In this talk, we introduce the various characterizations of flag Hardy spaces. These characterizations include maximal functions, the Littlewood-Paley square function, Lusin area integral, Riesz transforms and atomic decompositions.

(12/18 Fri. 14 : 10 — 14 : 40)

Muckenhoupt-Wheeden Conjectures from C.Z.O to Fractional operator

Chong-Wei Liang

National Taiwan University

Abstract: The starting point of this work goes back to 1971, when C.Fefferman and E.Stein,in order to establish vector-valued estimates for the maximal function,proved that the weak $(1,1)$ type estimate of maximal function. In this talk,we will introduce the weak $(1,1)$ type estimate of the Calderon -Zygmund operator,sparse operator and the fractional integral.

(12/18 Fri. 14 : 40 — 15 : 10)

Introduction to L_p Outer Measure Theory

Yu-Hsiang Lin

National Taiwan University

Abstract: In 1997, Thiele and Lacey proved the boundedness of bilinear Hilbert transform by time frequency analysis. After then, time frequency analysis has become a prevalent tool in harmonic analysis. In 2014, Do and Thiele discovered that we can unify some classical results such as Carleson embedding, paraproduct and time frequency analysis in the new language, L_p outer measure theory. In this talk, I will briefly introduce the history, motivation, and the mechanism of L_p outer measure theory and explain why it's natural to view these objects in this language.

Limit Theorems for Additive Functional of Diffusion Process

Te-Chun Wang

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Abstract

In the MCMC algorithm, it's natural to consider the asymptotic behavior of the additive functional for the recurrent linear diffusion X with reversible measure π . Let X be positive recurrent (i.e., π is the reversible probability measure). By the approximation of discrete-time Markov process, we know that

$$\frac{1}{t} \int_0^t f(X_s) ds \xrightarrow{t \rightarrow \infty} \int_{\mathbb{R}} f(x) \pi(x) dx \quad (a.s.),$$

but this limit theorem fails when X is null recurrent (i.e., π is sigma-finite measure). For example, if B is the standard Brownian motion with $B_0 = 0$, then

$$\frac{1}{t} \int_0^t f(B_s) ds \xrightarrow{t \rightarrow \infty} 0 \quad (a.s.),$$

In this paper, we prove that the local time $L_t(X)$ of X at level 0 is the accurate time scale for the strong law of large number (SLLN). That is, if X is a recurrent linear diffusion such that $X_0 = 0$, X is a solution of $dX_t = \sigma(X_t)dB_t + b(X_t)dt$, and π is a reversible measure of X , then

$$\frac{1}{L_t(X)} \int_0^t f(X_s) d\langle X, X \rangle_s \xrightarrow{t \rightarrow \infty} \int_{\mathbb{R}} f(x) \frac{\pi(x)}{\pi(0)} \frac{\sigma(x)^2}{\sigma(0)^2} dx \quad (a.s.).$$

For example, if B is the standard Brownian motion with $B_0 = 0$, then

$$\frac{1}{L_t(B)} \int_0^t f(B_s) ds \xrightarrow{t \rightarrow \infty} \int_{\mathbb{R}} f(x) dx \quad (a.s.).$$

Moreover, we also prove the concentration of the additive functional of Brownian motion (i.e., the central limit theorem)

$$\sqrt{L_t(B)} \left(\frac{1}{L_t(B)} \int_0^t f(B_s) ds - \int_{\mathbb{R}} f(x) dx \right) \xrightarrow{d} \mathcal{N}(0, \sigma_f^2),$$

where

$$\sigma_f^2 = \int_{\mathbb{R}} \int_{\mathbb{R}} f(x) f(y) 4(|x| \wedge |y|) 1_{xy \geq 0} dx dy.$$

(12/18 Fri. 16 : 00 — 16 : 30)

Introduction to free probability and free Brownian motion

Tsung-Yi Ma

National Taiwan University

Abstract: Free probability is analogue of classical probability theory, which is developed by Voiculescu around 1985. In the first part of this talk, I will introduce the basic concept of free probability, and give some examples. In second part, I will introduce the abstract denition of free Brownian motion and construct an example of it.

(12/18 Fri. 16 : 40 — 17 : 10)

Introduction to bi-freeness

Te-Lun Lu

National Taiwan University

Abstract: Dan-Virgil Voiculescu build the theory of free probability in 1985 in order to classify operator algebras. In 2000s, he released 4 papers that systematically characterize a new concept, bi-freeness, in free probability theory. In the first part of this talk, I will give a more abstract definition of freeness which is actually equivalent to the usual one. In the second part, I will generalize the concept in freeness, such as convolution, cumulant and central limit, to the bi-free case and provide some examples.